Eur. Phys. J. C 30, 135–143 (2003)
Digital Object Identifier (DOI) 10.1140/epjc/s2003-01258-5

THE EUROPEAN PHYSICAL JOURNAL C

Virtual black holes in generalized dilaton theories

D. Grumiller[1,a], W. Kummer[1,b], D.V. Vassilevich[2,c,*]

[1] Institut für Theoretische Physik, Technische Universität Wien, Wiedner Hauptstr. 8–10, 1040 Wien, Austria
[2] Max-Planck-Institut für Mathematik in den Naturwissenschaften, Inselstr. 22-26, 04103 Leipzig, Germany

Received: 7 August 2002 / Revised version: 2 June 2003 /
Published online: 11 July 2003 –

Abstract. The virtual black hole phenomenon, which has been observed previously in specific models, is established for generic 2D dilaton gravity theories with scalar matter. The ensuing effective line element can become asymptotically flat only for two classes of models; among them spherically reduced theories and the string inspired dilaton black hole. We present simple expressions for the lowest order scalar field vertices of the effective theory which one obtains after integrating out geometry exactly. Treating the boundary in a natural and simple way, asymptotic states, tree-level vertices and the tree-level S-matrix are conformally invariant. Examples are provided pinpointing the physical consequences of virtual black holes on the (CPT-invariant) S-matrix for gravitational scattering of scalar particles. For minimally coupled scalars the evaluation of the S-matrix in closed form is straightforward. For a class of theories including the string inspired dilation black hole all tree-graph vertices vanish, which explains the particular simplicity of that model and at the same time shows yet another essential difference to the Schwarzschild case.

1 Introduction

Recent years have seen remarkable progress in the quantum treatment of two dimensional models of gravity, so-called generalized dilaton theories (GDTs) (for a recent review cf. [1]), which include, most prominently, the Schwarzschild black hole (BH) and the string inspired CGHS model [2].

GDTs have been quantized in the last decade mostly by Dirac's canonical method [3] and, less frequently, by the path integral technique [4–8]. Indeed path integral quantization of a system with no propagating physical modes may appear to be something of an overkill, although both formalisms encounter essentially the same subtleties, albeit in different disguises.

However, once matter is switched on, the path integral approach to us appears to be superior. In addition to purely technical advantages the main reason is the much closer relation to genuine physical *observables*[1] like S-matrix elements. In fact, results like the ones to be discussed in our present note have been derived until now exclusively within this theoretical setup[2] [6, 7].

Especially as a consequence of the non-perturbative treatment of the geometric part, achieved in a "covariant Hamiltonian" action, together with a specific temporal gauge choice [4, 5], rather powerful new tools are available to tackle some persistent problems surrounding especially the Schwarzschild BH, but with consequences in other situations of general relativity which can be reduced effectively to a two dimensional problem[3].

Proceeding along the well established paths of quantum field theory, already at the level of the (non-local) vertices of matter fields, to be used in a systematic perturbative expansion in terms of Newton's constant, a highly non-trivial and physically intriguing phenomenon can be observed, namely the so-called "virtual black hole" (VBH). This notion originally has been introduced by Hawking [14], but in our recent approach the VBH for spherically

[a] e-mail: grumil@hep.itp.tuwien.ac.at
[b] e-mail: wkummer@tph.tuwien.ac.at
[c] e-mail: vassil@itp.uni-leipzig.de
[*] On leave from V. Fock Institute of Physics, St. Petersburg University, 198904 St. Petersburg, Russia

[1] By *observable* we mean "objects to be measured" rather than observables in the formal sense of Dirac. Of course the latter play a crucial role in Dirac quantization, but they are not necessarily identical to the former. Measuring the angular momentum of an electron (which is an observable in Dirac's sense) means to couple it to an external (magnetic) field and to calculate a corresponding S-matrix element which is an *observable* in the physical sense because it can be compared with experiments

[2] Of course, within the Dirac quantization approach since the seminal work of Kuchař on the quantization of gravitational waves [9] also interesting results with matter have been obtained, e.g. in the thin shell approach [10] where bound states might occur [11]

[3] At the quantum level the complications due to the "dimensional reduction anomaly" [12] may come into play, but reasonable results for specific questions like the Hawking flux from BHs [13] suggest that even these complications may be of limited importance

reduced gravity (SRG) emerges naturally in Minkowski signature space-time, without the necessity of additional ad hoc assumptions. The following observations are helpful to understand our notion of a VBH: the effective geometry that is obtained after taking into account the first non-trivial matter correction in our background independent quantization procedure contains a BH geometry localized on a light-like cut (see the Carter–Penrose diagram in Fig. 2 below). The non-trivial part of this geometry looks like a BH geometry in a Rindler background. This type of BH is not felt by the asymptotic states but only indirectly via scattering processes (which are encoded in the lowest order vertices to be calculated in this work). It can be traced back to a discontinuity in the geometric part of a conserved quantity (denoted by $\mathcal{C}^{(g)}$) which exists in all theories under consideration; see (3.14) below. Thus, whenever $\mathcal{C}^{(g)}$ has a discontinuity a VBH is present. The first discussion in [6] (SRG with in $D = 2$ minimally coupled scalars) was too simple to yield an interesting S-matrix, unless mass terms of the scalar field were added. The situation had improved for non-minimally coupled scalars [7] where the lowest order S-matrix indeed exhibited interesting features: forward scattering poles, monomial scaling with energy, CPT invariance, and pseudo-self-similarity in its kinematic sector [8].

In the present work we extend this analysis to arbitrary GDTs and show that the VBH phenomenon is generic. However, for one particular class of models, including the in the 2D-community well-known CGHS model, no tree-level scattering exists. Therefore the VBH phenomenon is not observable there (at least at the classical level). This result again pinpoints its special role among GDTs and provides a somewhat physical way to explain its simplicity. It also confirms that the Schwarzschild BH and the dilaton BH differ essentially.

In order to make this note self-contained we recapitulate in Sect. 2 some basic features of the path integral quantization. Section 3 is devoted to the determination of the vertices which are already non-local at the tree level, generalizing previous results on SRG [6,7] to generic dilaton theories (cf. (2.2) or (2.4) below). Although the consequences of these vertices for tree-level scattering of scalars under the influence of their own gravitational interactions is a classical phenomenon, their treatment from an S-matrix point of view is advantageous. Even without an explicit evaluation of special cases the general features can be discussed quite broadly (Sect. 4).

2 Two dimensional quantum gravity with matter

In its first order gravity (FOG) version the GDT action reads

$$L = L^{(\mathrm{FOG})} + L^{(m)}, \tag{2.1}$$

with the geometric part

$$L^{(\mathrm{FOG})} = \int_{\mathcal{M}_2} \left[X_a D \wedge e^a + X \mathrm{d} \wedge \omega + \epsilon \mathcal{V}(X^a X_a, X)\right]. \tag{2.2}$$

Geometry is expressed by the one form zweibeins e^a ($a = +, -$ in the local Lorentz frame using light-cone gauge) and the one form spin connection $\omega^a{}_b = \epsilon^a{}_b \omega$, appearing in the covariant derivative $D^a{}_b = \delta^a_b \mathrm{d} + \omega^a{}_b$ (the volume form is denoted by $\epsilon = \frac{1}{2}\epsilon_{ab} e^a \wedge e^b$). The dependence on the auxiliary fields X^a and the dilaton field X in the potential (using $X^a X_a = 2X^+X^-$)

$$\mathcal{V}(X^a X_a, X) = X^+X^- U(X) + V(X) \tag{2.3}$$

encodes the models relevant for our purposes. The action (2.2) is classically and quantum mechanically equivalent [5] to the more familiar general second order dilaton action

$$L^{(\mathrm{SOG})} = \int \mathrm{d}^2 x \sqrt{-g} \times \left[-X\frac{R}{2} - \frac{U(X)}{2}(\nabla X)^2 + V(X)\right]. \tag{2.4}$$

For the matter part we choose a (non-minimally coupled) massless scalar field:

$$L^{(m)} = \frac{1}{2} \int_{\mathcal{M}_2} F(X) \mathrm{d}\phi \wedge *\mathrm{d}\phi, \tag{2.5}$$

with an – in principle arbitrary – coupling function $F(X)$. In practice the cases $F = \text{const.}$ (minimal coupling) and $F \propto X$ (SRG) are the most relevant ones.

The path integration can be performed using e.g. a “temporal gauge”

$$\omega_0 = 0, \quad e_0^- = 1, \quad e_0^+ = 0. \tag{2.6}$$

It is convenient to introduce comprehensive notations (in accordance with [1]) for the remaining geometric variables:

$$(q_1, q_2, q_3) := (\omega_1, e_1^-, e_1^+), \\ (p_1, p_2, p_3) := (X, X^+, X^-). \tag{2.7}$$

In terms of (2.7) the action (2.1) can be rewritten as

$$L = \int \mathrm{d}^2 x \Big[p_i \dot{q}_i + q_1 p_2 - q_3 \left(V(p_1) + U(p_1) p_2 p_3\right) \\ + F(p_1)\left((\partial_1 \phi)(\partial_0 \phi) - q_2 (\partial_0 \phi)^2\right)\Big], \tag{2.8}$$

where the gauge condition (2.6) has been taken into account. The abbreviations

$$\varPhi_0 := \frac{1}{2}(\partial_0 \phi)^2, \qquad \varPhi_1 := \frac{1}{2}(\partial_0 \phi)(\partial_1 \phi). \tag{2.9}$$

are suggested by (2.8).

The action (2.8) is linear in q_i. This is also true for the matter action (2.5) when it is expressed in terms of the variables (2.7) and for the source terms for q_i which are neither included explicitly in (2.8) nor in what follows. Therefore, the integration over q_i can be performed exactly – without the necessity of introducing a split of geometry into “background” and “fluctuations” – yielding three functional δ-functions $\delta(p_i - p_i^{\mathrm{cl}})$, where p_i^{cl} are

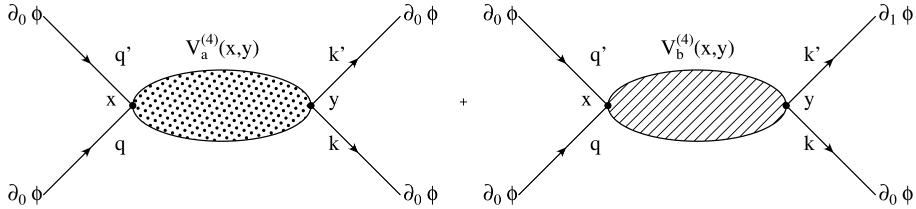

Fig. 1. The total $V^{(4)}$-vertex (with outer legs) contains a symmetric contribution $V_a^{(4)}$ and (for non-minimal coupling) a non-symmetric one $V_b^{(4)}$. The shaded blobs depict the intermediate interactions with VBHs

solutions of the classical equations of motion for p_i in the presence of external sources. For each specific model the $p_i^{\rm cl}$ are given functions of Φ_0 and Φ_1. The three delta functions are then used to integrate p_i. Retaining only the dependence on the source σ of the scalar field the simplified generating functional of the Green functions reads[4]

$$W(\sigma) = \int (\mathcal{D}\phi) \exp \mathrm{i} L^{\rm eff}, \tag{2.10}$$

$$L^{\rm eff} = \int \mathrm{d}^2 x \left[2F(p_1^{\rm cl})\Phi_1 + \tilde{\mathcal{L}}(p_i^{\rm cl}) + \sigma\phi \right], \tag{2.11}$$

$$\tilde{\mathcal{L}}(p_1^{\rm cl}) = -\tilde{g} w'(p_1^{\rm cl}), \tag{2.12}$$

where

$$w(p_1) := \int^{p_1} I(y)V(y)\mathrm{d}y, \tag{2.13}$$

with

$$I(p_1) := \exp \int^{p_1} U(y)\mathrm{d}y. \tag{2.14}$$

Here $\tilde{g}$ plays the role of an effective coupling. It will be fixed below establishing the unit of length. Note that $p_1^{\rm cl}$ depends non-locally and non-polynomially on Φ_0. In the absence of external sources it is determined by the classical equations of motion (e.o.m.) (3.3) and (3.4) below.

Up to this point all path integrals have been exact. In the next step the quadratic terms in ϕ, respectively the linear expressions in Φ_0 and Φ_1, are isolated in (2.11). Terms of higher order in ϕ are interpreted as vertices in a standard perturbation theory.

A peculiar property of 2D gravity theories is the presence of an "absolute" (in space and time) conservation law $\mathrm{d}\mathcal{C} = \mathrm{d}(\mathcal{C}^{(g)} + \mathcal{C}^{(m)}) = 0$ [15] even when matter is taken into account [16]. Its geometric part only depends on the auxiliar fields, respectively p_i (cf. (2.7)). The (classical) expression for the latter,

$$\mathcal{C}^{(g)} := I(p_1)p_2p_3 + w(p_1), \tag{2.15}$$

will be needed below with I of (2.14) playing the role of an integrating factor. $\mathcal{C}^{(g)}$ is related to the so-called "mass-aspect function" [17] and becomes simply proportional to the ADM mass for asymptotically flat metrics.

It should be noted that in models (2.4) which are connected by a conformal transformation

$$g_{\mu\nu} = \mathrm{e}^{2\rho(X)}\tilde{g}_{\mu\nu}, \qquad U(X) = \tilde{U}(X) + 2\rho'(X), \\ V = \mathrm{e}^{-2\rho(X)}\tilde{V}(X), \tag{2.16}$$

the combination $w(p_1) = \tilde{w}(p_1)$ as defined in (2.13) is invariant.

[4] In the present context questions regarding the measure, back reactions and geometric source terms are irrelevant. Therefore, the generating functional of the Green functions simplifies considerably as compared to the exact case [1,5,7]

3 Construction of the lowest order tree-graphs

In the present paper we are interested in the tree-level amplitudes and in the generic properties of ensuing S-matrix elements. The final result can be formulated in a rather intuititive way as explained below. A more rigorous derivation can be found in our previous work [1,6,7].

The lowest order vertex from (2.12) is quartic in ϕ. Thus in the notations (2.9) the generic four-point interaction term reads

$$V^{(4)} = \int \mathrm{d}^2x\mathrm{d}^2y \Big[V_a^{(4)}(x,y)\Phi_0(x)\Phi_0(y) \\ + V_b^{(4)}(x,y)\Phi_0(x)\Phi_1(y) \\ + V_c^{(4)}(x,y)\Phi_1(x)\Phi_1(y) \Big] \tag{3.1}$$

with some kernels $V_{a,b,c}^{(4)}(x,y)$. As $p_1^{\rm cl}$ is independent of Φ_1 (cf. (3.3) and (3.4) below) it is clear that the interaction term with $V_c^{(4)}$ never appears in $V^{(4)}$ which is graphically represented in Fig. 1.

In principle $V^{(4)}$ could be abstracted directly from (2.11). However, the expressions $p_i^{\rm cl} = p_i^{\rm cl}(\Phi_0)$ require the solution of three coupled first order differential equations which are nothing else than the classical e.o.m. (3.3)–(3.5) below. The geometric quantities q_i could be obtained as expectation values. Also for them it is much preferable to go back to their (classical) e.o.m. Then, for example, to calculate $V_a^{(4)}(x,y)$, one may use the following recipe [1,5–7]. With the matter distribution localized at a point y,

$$\Phi_i(x) = c_i\delta^2(x-y), \qquad i = 0,1, \tag{3.2}$$

one computes solutions $q_i(x;c_i,y)$, $p_i(x;c_i,y)$ of the classical e.o.m. with the matter sources given by (3.2). Recalling that the coefficient in front of Φ_0 in (2.8) is $[-2F(p_1)q_2]$, $V_a^{(4)}(x,y)$ simply is given by $[-2F(p_i(x;c_i,y))q_2(x;c_i,y)]$, taken at linear order of c_0. The vertex $V_b^{(4)}(x,y)$ can be constructed in a similar manner. In fact, for all tree-graph vertices (with $2N$ outer legs) the knowledge of the solutions of the classical e.o.m. with localized matter sources (at $N-1$ different points) is sufficient.

3.1 Equations of motion

The gauge-fixed action (2.8) yields the following e.o.m.:

$$\partial_0 p_1 = p_2\,, \tag{3.3}$$
$$\partial_0 p_2 = -2F(p_1)\Phi_0\,, \tag{3.4}$$
$$\partial_0 p_3 = -V(p_1) - p_2 p_3 U(p_1)\,, \tag{3.5}$$
$$\partial_0 q_1 = q_3\left(V'(p_1) + p_2 p_3 U'(p_1)\right) -2F'(p_1)(\Phi_1 - q_2\Phi_0)\,, \tag{3.6}$$
$$\partial_0 q_2 = -q_1 + q_3 p_3 U(p_1)\,, \tag{3.7}$$
$$\partial_0 q_3 = q_3 p_2 U(p_1)\,. \tag{3.8}$$

The system (3.3)–(3.8) can be solved exactly for $\Phi_0 = \Phi_1 = 0$, as well as for localized matter (3.2). Then two patches ($x^0 < y^0$ and $x^0 > y^0$) must be matched by appropriate conditions at $x^0 = y^0$. The structure of (3.3)–(3.8) shows that p_1, p_3, q_2, q_3 are continuous, whereas p_2, q_1 may jump at $x^0 = y^0$. This yields six conditions relating the "asymptotic" integration constants ($x^0 > y^0$) to the "inner" ones ($x^0 < y^0$).

The conditions for the determination of the six asymptotic integration constants are as follows.
(1) $p_1|_{x^0\to\infty} = x^0$ fixes two integration constants. It implies $p_2|_{x^0\to\infty} = 1$.
(2) $\mathcal{C}^{(g)}\big|_{x^0\to\infty} = \mathcal{C}_\infty$ by (2.15) yields the constant in p_3. We will choose $\mathcal{C}_\infty = 0$, which corresponds to the ground state – e.g. for SRG which is asymptotically flat this condition yields Minkowski space-time.
(3) $q_3|_{x^0\to\infty} = 1\cdot I(p_1)$ establishes the unit of length.
(4) The remaining two integration constants are called m_∞ and a_∞, because for SRG they correspond to a Schwarzschild and a Rindler term, respectively. Both of them enter $q_2|_{x^0\to\infty}$.

It should be noted that not all six integration constants are independent: (3.3)–(3.8) are not the complete set of (classical) e.o.m. of the model. Without gauge fixing also equations from variation of $e_0^\pm$ and ω_0 – the secondary constraints in the Hamiltonian formalism [1] – must hold. In the quantum formalism they appear as "Ward identities" and resemble (3.3) and (3.4), but with the ∂_1 derivative replacing ∂_0. It turns out that they imply two additional independent relations,

$$\mathcal{C}_\infty = m_\infty\,, \qquad a_\infty = 0\,. \tag{3.9}$$

leaving the only non-trivial asymptotic quantity to be m_∞. Due to our conventions m_∞ is negative for a positive BH mass.

Fixing these integration constants in this manner the global solutions for the momenta from (3.3)–(3.5) to $\mathcal{O}(c_0)$ are

$$p_1 = x^0 + 2F(y^0)(x^0 - y^0)h_0\,, \tag{3.10}$$
$$p_2 = 1 + 2F(y^0)h_0\,, \tag{3.11}$$
$$p_3 = I^{-1}(p_1)\left(m_\infty - 2F(y^0)w(y^0)h_0 - \frac{1}{p_2}w(p_1)\right)\,, \tag{3.12}$$

with the abbreviations

$$h_i = c_i\theta(y^0 - x^0)\delta(x^1 - y^1)\,, \qquad i = 0, 1\,. \tag{3.13}$$

From now on all quantities will be given only to the required linear order in c_0 and c_1. The geometric part of the conserved quantity from (2.15) becomes

$$\mathcal{C}^{(g)} = m_\infty + 2F(y^0)\left(m_\infty - w(y^0)\right)h_0\,. \tag{3.14}$$

Thus, m_∞ fixes the asymptotic value of $\mathcal{C}^{(g)}$. One can observe already the VBH phenomenon: The discontinuity in h_0 at $x^0 = y^0$ also carries over to $\mathcal{C}^{(g)}$. Thus the VBH is a generic feature of *all* GDTs.

The solution for the component q_1 of the spin connection is not needed for the line element and only $\partial_0 q_1$ (which can be read off from (3.6)) is needed for the curvature scalar[5]. The line element in the gauge in (2.6) from (2.7) only depends on the zweibeins q_2, q_3 which are the solutions of (3.6) and (3.7):

$$q_2 = m_\infty - w(p_1) - 2(x^0 - y^0)F'(y^0)h_1 + \Big[4F(y^0)\left(w(x^0) - w(y^0)\right) -2(x^0 - y^0)\,(Fw)'\big|_{y^0}\Big]h_0\,, \tag{3.15}$$
$$q_3 = I(p_1)\,. \tag{3.16}$$

3.2 Effective geometry

The line element $(\mathrm{d}s)^2 = 2e^+\otimes e^-$ in the gauge (2.6) reads

$$(\mathrm{d}s)^2 = 2q_3\mathrm{d}x^0\mathrm{d}x^1 + 2q_2 q_3(\mathrm{d}x^1)^2\,. \tag{3.17}$$

In order to bring it into a more familiar form we define a "radial" variable

$$\mathrm{d}r = b q_3(x^0)\mathrm{d}x^0\,, \qquad b \in \mathbb{R}^*\,, \tag{3.18}$$

and a "null coordinate"

$$\mathrm{d}u = b^{-1}\mathrm{d}x^1\,, \tag{3.19}$$

thus obtaining the line element in Sachs–Bondi form

$$(\mathrm{d}s)^2 = 2\mathrm{d}r\mathrm{d}u + K(r, u; r_0, u_0)(\mathrm{d}u)^2\,, \tag{3.20}$$

with the "Killing norm"[6] $K(r, u; r_0, u_0) = 2b^2 q_2 q_3$.

More explicitly K reads

$$\begin{aligned} K(r, u; r_0, u_0) &= K_\infty\left[1 + 2F(y^0)U(x^0)(x^0 - y^0)h_0\right] \\ &\quad -4b^2 I(x^0)(x^0 - y^0) \\ &\qquad \times\left[F(y^0)\left(V(x^0)I(x^0) + V(y^0)I(y^0)\right)h_0 + F'(y^0)\left(w(y^0)h_0 + h_1\right)\right] \\ &\quad +8b^2 I(x^0)F(y^0)\left(w(x^0) - w(y^0)\right)h_0\,, \end{aligned} \tag{3.21}$$

with

$$K_\infty = 2b^2 I(x^0)\left(m_\infty - w(x^0)\right)\big|_{c=0}\,. \tag{3.22}$$

[5] For SRG the curvature scalar can be calculated more elegantly using the Kerr–Schild decomposition [8]

[6] The quantities (r_0, u_0) are related to (y^0, y^1) like (r, u) to (x^0, x^1)

Its continuity at $x^0 = y^0$ is manifest in (3.21). For SRG in the VBH region a Schwarzschild term (proportional to $1/r$) and a Rindler term (proportional to r) are present in (3.21). Their appearance is somewhat surprising because apart from fixing the asymptotic boundary values of all geometric quantities we have made no assumption whatsoever on the geometry, except for its (Minkowskian) signature.

Depending on $I(x^0)$ and $w(x^0)$ asymptotic flatness[7] can be achieved for which there are two possibilities.
(1) $I(x^0)w(x^0)\big|_{c=0} = \text{const.} \in \mathbb{R}^*$ and $\lim_{r\to\infty} I(r) \propto r^\alpha$, with $\alpha \leq 0$.
(2) $I(x^0) = \text{const.} \in \mathbb{R}^*$ and $\lim_{r\to\infty} w(r) \propto r^\alpha$, with $\alpha \leq 0$.

The first scenario is fulfilled for all SRG models arising by reduction from D dimensions ($F \propto X$, $x^0 \propto r^{D-2}$, $D > 3$, $\lambda \in \mathbb{R}^*$)

$$\begin{aligned} U(X) &= -\frac{(D-3)}{(D-2)X}\,, \\ V(X) &= -\frac{\lambda^2}{2}(D-2)(D-3)X^{(D-4)/(D-2)}\,, \qquad (3.23) \\ I(X) &= X^{(3-D)/(D-2)}\,, \\ w(X) &= -\frac{\lambda^2}{2}(D-2)^2 X^{(D-3)/(D-2)}\,, \qquad (3.24) \end{aligned}$$

including the CGHS model as the formal limiting case $D \to \infty$, $F = \text{const.}$ and $\lambda \propto 1/D$ [1]. The second case applies e.g. when SRG models are transformed conformally so that $\tilde{U} = 0$.

For GDTs the only independent physical geometric quantity is the curvature scalar[8] $R^{(\text{VBH})} = 2(e)^{-1}\tilde{\epsilon}^{\alpha\beta}\partial_\alpha\omega_\beta = -2q_3^{-1}\partial_0 q_1$ which in terms of the solutions (3.16) and (3.6) becomes

$$\begin{aligned} R^{(\text{VBH})} = &-2\left(V'(p_1) + p_2 p_3 U'(p_1)\right) \qquad (3.27) \\ &+4(I(p_1))^{-1}F'(p_1)\left(\Phi_1 - q_2\Phi_0\right)\,. \end{aligned}$$

[7] Statements about asymptotic flatness are in general *not* invariant under conformal transformations because in addition to w-dependence we have dependence on the boundary value m_∞ and also dependence on I, a conformally non-invariant quantity.
For definiteness we assume that the asymptotic region corresponds to $x^0, r \to \infty$

[8] There is a slight subtlety at this point: in conformal frames where $I \neq 1$ the curvature scalar (as defined by the Hodge dual of the curvature two form) is not just the second derivative of the Killing norm, because part of the geometric information is encoded in the torsion. In the absence of matter one obtains for the curvature associated with the Levi-Civita connection

$$\tilde{R} = 2\left(V'(p_1) + p_2 p_3 U'(p_1)\right) - 4\frac{w''(p_1)}{I(p_1)}\,, \qquad (3.25)$$

while the Hodge dual of the curvature two form reads

$$R = -2\left(V'(p_1) + p_2 p_3 U'(p_1)\right)\,. \qquad (3.26)$$

In the absence of torsion ($U = 0$) both expressions coincide. The torsion two form T^a is proportional to the volume two form times $U(p_1)t^a$ with $t^+ = I(p_1)$ and $t^- = \mathcal{C}^{(g)} - w(p_1)$

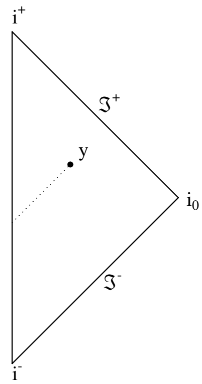

Fig. 2. CP diagram of the VBH

One can read off from (3.27) that the first term provides a step function at $x^0 = y^0$ while the second term yields a contribution[9] $\delta(y^0 - x^0)$. The latter is absent for minimal coupling. If additionally $U' = 0$ – as for instance in the Katanaev–Volovich model [19] – the curvature scalar even becomes continuous. As the curvature depends on U and V separately it is quite different in conformally related theories.

The Carter–Penrose (CP) diagram of the VBH geometry is analogous to the SRG case discussed in [8]: there will be a light-like cut where the non-trivial part of geometry (namely the VBH) is localized with δ-like contributions to the curvature scalar on its end-points. The rest of the CP diagram is determined by (3.22). For instance, if the potentials and the asymptotic mass m_∞ are chosen such that K_∞ is constant, then the CP diagram is given by the one for Minkowski space beside the VBH. One can simply determine the full CP diagram by disregarding the VBH in a first step (thus drawing an ordinary CP diagram for a line element (3.20) with K being replaced by K_∞) and then simply adding the light-like cut representing the VBH.

As in the previous case one obtains a family of CP diagrams labelled by the end-point (denoted by y in Fig. 2). All these (off-shell) geometries are (continuously) summed in the S-matrix.

3.3 Symmetric four-point vertex

To obtain the interaction vertex $V_a^{(4)}$ one has to substitute[10] the classical solutions (3.10)–(3.16) in the interaction term $-F(p_1)q_2\Phi_0(x)$, truncate it to linear order in c_0, multiply by $\Phi_0(y)$, and integrate over the space-time. We

[9] In some models (e.g. all SRG models) an additional (δ-like) contribution at the "origin" $x^0 = 0$ may arise [18]

[10] Especially at this point it might be convenient to consult Sect. 8 of the review in [1] or the corresponding parts of the original papers [5–7]

have

$$V_a^{(4)} = -2\int_x\int_y \Phi_0(x)\Phi_0(y)\theta(x^0-y^0)\delta(x^1-y^1)F(x^0)F(y^0) \times\Big[4\left(w(x^0)-w(y^0)\right)-2(x^0-y^0)\Big(w'(x^0)+w'(y^0) +\tfrac{F'(y^0)}{F(y^0)}w(y^0)+\tfrac{F'(x^0)}{F(x^0)}\left(w(x^0)-m_\infty\right)\Big)\Big]. \quad (3.28)$$

This vertex has the following properties.
(1) It vanishes for $x^0=y^0$.
(2) It depends only on the combination (2.13) of U and V in the function $w(p_1)$; since $w(p_1)$ is invariant under conformal transformations also the vertex is invariant, if m_∞ was fixed to the same value in all conformal frames.
(3) It respects the $\mathbb{Z}_2$ symmetry $F(p_1)\to -F(p_1)$; thus, a change of the sign of the gravitational coupling constant (implicitly contained in $F(p_1)$) does not change the result for this vertex[11].

3.4 Non-symmetric four-point vertex

An additional vertex arises for non-minimal coupling ($F'\neq 0$ in (2.5)):

$$V_b^{(4)} = -4\int_x\int_y \Phi_0(x)\Phi_1(y)\delta(x^1-y^1)F(x^0)F'(y^0)\left|x^0-y^0\right| . \quad (3.29)$$

It shares the properties with its symmetric counterpart, except for the fact that it is even independent of U and V.

No further vertices appear, unless one adds mass-terms (they yield additional vertices as shown in [6]) or local self-interactions (which are rather trivial).

4 S-matrix elements

The S-matrix element with ingoing modes q, q' and outgoing ones k, k',

$$T(q,q';k,k') = \frac{1}{2}\left\langle 0\left|a_k^- a_{k'}^-\left(V_a^{(4)}+V_b^{(4)}\right)a_q^+a_{q'}^+\right|0\right\rangle , \quad (4.1)$$

depends not exclusively on the vertices (3.28) and (3.29), but also on the asymptotic states of the scalar field with corresponding creation/anihilation operators $a^\pm$ obeying canonical commutation relations of the form $\left[a_k^-,a_{k'}^+\right]\propto\delta(k-k')$.

Their explicit form is model dependent and sensitive to m_∞. That is why from now on we restrict ourselves to general statements rather than calculations which have already appeared to be rather lengthy in previous work on SRG [7].

[11] It is interesting to note that the one-loop quantum effective action for non-minimally coupled matter fields respects a different $\mathbb{Z}_2$ symmetry [20]: $F(p_1)\to F(p_1)^{-1}$

4.1 Asymptotic matter states

The metric extracted from $(\mathrm{d}s)^2$ in (3.17) at $x^0\to\infty$ determines the asymptotic matter states in the S-matrix. They not only depend on the model but also on the value of m_∞. In this limit the Klein–Gordon equation, expressed by the asymptotic form of $q_2=m_\infty-w(x^0)$ reads

$$\partial_1\partial_0\phi+\frac{F'(x^0)}{2F(x^0)}\partial_1\phi = q_2\partial_0\partial_0\phi+q_2\frac{F'(x^0)}{F(x^0)}\partial_0\phi-w'(x^0)\partial_0\phi . \quad (4.2)$$

If (4.2) can be solved exactly and if the set of solutions is complete and normalizable (in an appropriate sense) a Fock space for incoming and outgoing scalars can be constructed in the usual way. Equation (4.2) is conformally invariant with the previous *caveat* concerning m_∞. Since the integration constant m_∞ enters (4.2) the asymptotic states depend on the choice of boundary conditions.

Most string inspired dilaton models exclusively use minimally coupled scalars ($F(p_1)=$ const.). In that case (4.2) simplifies drastically:

$$\partial_0(\partial_1\phi-q_2\partial_0\phi)=0 . \quad (4.3)$$

Solutions of this equation are outgoing modes:

$$\phi_{\text{out}}=a_k^+\exp\left(\mathrm{i}kx^1\right), \quad (4.4)$$

and ingoing ones:

$$\phi_{\text{in}}=a_k^-\exp\left(-\mathrm{i}k\left(x^1+\int^{x^0}\frac{\mathrm{d}z}{m_\infty-w(z)}\right)\right). \quad (4.5)$$

Using first (3.18) and (3.19) and then the Regge–Wheeler redefinition $\mathrm{d}t=\mathrm{d}u+\mathrm{d}r/K_\infty(r;r_0)$ with K_∞ given by (3.22) one can switch to the more transparent (r,t) coordinates. In terms of these we obtain ordinary asymptotic plane waves. After defining a properly normalized commutation relation between a_k^- and a_k^+ one can build the Fock space and calculate the S-matrix using the vertex (3.28) in complete analogy to [6] where the special case $m_\infty=0$, $U(p_1)=-1/(2p_1)$ and $V(p_1)=$ const. has been investigated.

We emphasize that only one vertex is present in the minimally coupled case. For the model considered in [6] this has led to the conclusion that one of the following three alternatives must hold: 1. the scattering amplitude diverges, 2. the scattering amplitude vanishes (if the virtual black hole is plugged by an ad hoc regularity condition) or 3. the scalar field acquires a mass term, thus producing a second four-point vertex.

Considering non-minimally coupled scalars induces two important complications: firstly, the asymptotic Klein–Gordon equation (and hence also the asymptotic states) differs, and secondly, an additional vertex (3.29) exists. For SRG especially the second drawback turned into a virtue, since the scattering amplitude exhibited some very nice features (to start with, it was finite without additional regularity conditions as opposed to the minimal case) [7,8,21].

4.2 CPT invariance

Since our effective theory is non-local, CPT invariance is not guaranteed by the CPT theorem[12] [22]. Therefore, possibilities of CPT violation must be explored, because they could imply a preferred direction of time.

Indeed, the result obtained for SRG in [8] can be generalized straightforwardly to arbitrary dilaton models. There is, however, a subtlety: in those previous calculations $m_\infty = 0$ was a natural consequence of the asymptotically flat metric. Under this assumption, the gauge choice $\omega_0 = 0$, $e_0^- = -1$, $e_0^+ = 0$ led to some trivial sign changes in intermediate formulae (in particular, e_1^- flipped its sign) and an overall sign change in the (real) scattering amplitude. From this and the fact that C and P act trivially, CPT invariance could be established.

In the presence of a non-vanishing integration constant m_∞ the question arises of how to relate its values for different gauges. There is only one way to retain CPT invariance: the new value for m_∞ must be related to the old one by $m_\infty^{CPT} = -m_\infty$. Then e_1^- (and also the vertices and the amplitude) again simply flip their sign.

The physical interpretation of this treatment of the integration constant is model dependent. For asymptotically flat models it means that the space-time and its mirror version must have the same ADM mass. For instance, in SRG the term $1 - 2m_\infty/r$ present in the Killing norm has to change into $-1 + 2(-m_\infty^{CPT})/r$ and thus $m_\infty^{CPT} = -m_\infty$; note that the ADM mass is $+m_\infty$ in the first case and $-m_\infty^{CPT}$ in the second case, so (by construction) it does not change. For generic other models no such interpretation seems to be available.

4.3 Conformal invariance

4.3.1 General considerations

As can be seen directly from (3.27) the curvature scalar obviously is not invariant under conformal transformations. For GDTs without matter (but with coupling to test particles because otherwise geometry is void[13]) this leads to the immediate conclusion that GDTs are *not* conformally invariant – for instance, the global structure can change by conformal transformations because in general they have one or more singular points (cf. e.g. [23] and references therein).

In the present context, however, we do not need test-particles anymore, because we have (scalar) matter available to test geometry, e.g. by preparing some initial scalar field configuration and measuring the final one[14]. The S-matrix describes the physical content of scattering processes.

Since both the asymptotic states and the vertices depend only on w (a conformally invariant combination of the potentials U and V), F (the coupling function, which by assumption is not changed by conformal transformations), and m_∞ (the asymptotic mass-scale, which can be fixed to the same numerical value in each conformal frame) we can conclude that at tree level to lowest order in the scalar field conformal invariance exists. Classically one can trivially extend this result to the non-perturbative level (cf. (2.11) and (2.12) which only depend on $w'(p_1)$), but *not* to one-loop level because the conformal anomaly will destroy this invariance.

4.3.2 The special role of CGHS

In the CGHS model ($U_{\text{CGHS}} = -1/X$, $V_{\text{CGHS}} = -2\lambda^2 X$, $F_{\text{CGHS}} = \text{const.}$) the derivative of the quantity (2.13) is constant. Therefore, according to (3.28) $V^{(4)}$ vanishes. Actually the absence of classical scalar vertices, to be used in perturbation theory, extends to arbitrary orders in Φ_0 and Φ_1 of (2.9) as can be verified by inspection of the Lagrangian (2.11) with (2.12). Thus, non-trivial scattering with an arbitrary number of external scalar legs would have to result from higher order quantum effects, to be derived e.g. from quantum back reaction which has not been considered in the simplified path integral (2.10). The absence of classical scattering from the string inspired CGHS model explains its simplicity and, at the same time, questions its status as an emblematic BH laboratory relative to all other GDTs (including especially SRG itself) where $w'(p_1) \neq \text{const.}$

Obviously this "scattering triviality" extends to a wider class of GDTs with minimally coupled scalars: if the potentials V_{ST} and U_{ST} in (2.2) or (2.4) are related by

$$U_{ST}(p_1)V_{ST}(p_1) + V'_{ST}(p_1) = 0 \tag{4.6}$$

the crucial quantity w'_{ST} remains constant. As an example of such models the "*ab*-family" $U = -a/X$, $V \propto X^{(a+b)}$ [25] can be considered. The triviality condition (4.6) establishes $b = 0$, containing the class of soluble models studied by Fabbri and Russo [26]. As can be seen from Fig. 3.12 in [1] the value $b = 0$ is a critical one, for it separates different "phases" of Carter–Penrose diagrams.

4.3.3 Presence of the VBH phenomenon

The VBH effect was tied to the appearance of a discontinuity in $\mathcal{C}^{(g)}$ of (3.14). Again the conformally invariant quantity w, here combined with the constant m_∞, plays a central role. By the same token as above the VBH is conformally invariant.

Despite the presence of the VBH in the CGHS model we register the peculiar situation that it has no (classically) observable consequences due to "scattering triviality"; see (4.6). Thus, scattering non-triviality implies the

[12] The parent dilaton gravity theory is local but contains singular interactions

[13] Geometry without test-particles to probe it is like the fiction of "empty space" or the fiction of quantum mechanical "observables" without external sources to probe them [23]

[14] We note parallels to the concept of "quantum singularities", where test-particles are replaced by a wavefunction and singularities can be smoothed out [24]

VBH phenomenon but not vice versa. Only for "generalized teleparallel[15]" theories with $w(p_1) = w_0 = \text{const.}$ the VBH phenomenon will be absent provided $m_\infty = w_0$.

5 Conclusion and outlook

We have established the existence of the virtual black hole phenomenon for all generalized dilaton theories with (scalar) matter, with the notable exception of "generalized teleparallel" theories. The corresponding effective geometry (3.20) shows essentially the same features as the previously discussed case of spherically reduced gravity [6–8]. The ensuing tree-level S-matrix for scattering of scalars is CPT and conformally invariant. For the CGHS model and a class of similar ones (4.6) where $w'(p_1)$ as defined in (2.13) is constant no tree-level vertices with any number of scalar legs are produced.

One can apply our methods to evaluate the S-matrix for all models where the set of solutions of (4.2) is complete and normalizable, i.e. where an asymptotic Fock space can be constructed. In some cases the results will be not very illuminating [6], in other cases they exhibit interesting properties [7, 8]. For instance, spherically reduced gravity from arbitrary dimension D could be worthwhile studying, since on the one hand for $D = 4$ we already know the result to be highly non-trivial and it would be rewarding to verify whether this is generic or rather a special feature of $D = 4$. On the other hand the formal limit $D \to \infty$ will yield the CGHS model with *non-minimally* coupled scalars. Moreover, for $m_\infty = 0$ the Fock space construction is straightforward in these particular cases, the asymptotic modes being D-dimensional s-waves.

Although some of our statements like tree-level conformal invariance and "scattering triviality" in certain models are valid for tree vertices with any number of scalar legs, we were concerned mostly with the lowest order of these vertices ((3.28) and (3.29)). The non-symmetric one vanishes for minimal coupling $F(p_1) = \text{const.}$

The next obvious step at tree level is a generalization to arbitrary six-point vertices. They will be the first ones to break the $\mathbb{Z}_2$ symmetry discussed in the paragraph below (3.28). Loop calculations, which will give insight into the nature of the information paradox are in progress. First results for the CGHS model are already available: the otherwise trivial specific heat becomes mass dependent at one-loop level [29].

We conclude this paper by noting that virtual black holes may be significant already at the collider energies according to some models (cf. e.g. [27]). In principle, our analysis may be extended even to the creation of real black holes at future colliders [28]. However, in a theoretical analysis one would have to give up spherical symmetry, so that a direct application of our approach would no longer be possible.

[15] Teleparallel theories obey $U = \text{const.}$, $V = 0$ while "generalized" refers to arbitrary $U(p_1)$ but retaining $V = 0$

Acknowledgements. This work has been supported by project P-14650-TPH of the Austrian Science Foundation (FWF) and the MPI MIS (Leipzig). We thank P. Fischer for discussion. D.G. is grateful to D. Konkowski for drawing his attention to the concept of quantum singularities.

References

1. D. Grumiller, W. Kummer, D.V. Vassilevich, Phys. Rept. **369**, 327, 429 (2002), hep-th/0204253
2. C.G. Callan, Jr., S.B. Giddings, J.A. Harvey, A. Strominger, Phys. Rev. D **45**, 1005 (1992), hep-th/9111056
3. D. Cangemi, R. Jackiw, Phys. Rev. Lett. **69**, 233 (1992), hep-th/9203056; A. Mikovic, Phys. Lett. B **291**, 19 (1992), hep-th/9207006; T. Thiemann, H.A. Kastrup, Nucl. Phys. B **399**, 211 (1993), gr-qc/9310012; J. Gegenberg, G. Kunstatter, Phys. Rev. D **47**, 4192 (1993), gr-qc/9302006; D. Cangemi, R. Jackiw, Ann. Phys. **225**, 229 (1993), hep-th/9302026; Phys. Lett. B **299**, 24 (1993), hep-th/9210036; A. Mikovic, Phys. Lett. B **304**, 70 (1993), hep-th/9211082; D. Louis-Martinez, J. Gegenberg, G. Kunstatter, Phys. Lett. B **321**, 193 (1994), gr-qc/9309018; D. Cangemi, R. Jackiw, Phys. Lett. B **337**, 271 (1994), hep-th/9405119; T. Strobl, Phys. Rev. D **50**, 7346 (1994), hep-th/9403121; K.V. Kuchař, Phys. Rev. D **50**, 3961 (1994), gr-qc/9403003; further references can be found in [1]
4. F. Haider, W. Kummer, Int. J. Mod. Phys. A **9**, 207 (1994); W. Kummer, H. Liebl, D.V. Vassilevich, Nucl. Phys. B **493**, 491 (1997), gr-qc/9612012; Nucl. Phys. B **513**, 723 (1998), hep-th/9707115
5. W. Kummer, H. Liebl, D.V. Vassilevich, Nucl. Phys. B **544**, 403 (1999), hep-th/9809168
6. D. Grumiller, W. Kummer, D.V. Vassilevich, Nucl. Phys. B **580**, 438 (2000), gr-qc/0001038
7. P. Fischer et al., Phys. Lett. B **521**, 357 (2001), gr-qc/0105034; Erratum B **532**, 373 (2002); D. Grumiller, Quantum dilaton gravity in two dimensions with matter, PhD thesis, Technische Universität Wien, 2001, gr-qc/0105078; P. Fischer, Vertices in spherically reduced quantum gravity, Master's thesis, Vienna University of Technology, 2001
8. D. Grumiller, Class. Quant. Grav. **19**, 997 (2002), gr-qc/0111097
9. K. Kuchař, Phys. Rev. D **4**, 955 (1971)
10. P. Hajicek, C. Kiefer, Nucl. Phys. B **603**, 531 (2001), hep-th/0007004; P. Hajicek, Nucl. Phys. B **603**, 555 (2001), hep-th/0007005; P. Hajicek, I. Kouletsis, gr-qc/0112060; gr-qc/0112061
11. I. Kouletsis, P. Hajicek, gr-qc/0112062
12. V. Frolov, P. Sutton, A. Zelnikov, Phys. Rev. D **61**, 024021 (2000), hep-th/9909086; P. Sutton, Phys. Rev. D **62**, 044033 (2000), hep-th/0003290; R. Balbinot et al., Phys. Rev. D **63**, 084029 (2001), hep-th/0012048; G. Cognola, S. Zerbini, Nucl. Phys. B **602**, 383 (2001), hep-th/0008061
13. W. Kummer, D.V. Vassilevich, Annalen Phys. **8**, 801 (1999), gr-qc/9907041
14. S.W. Hawking, Phys. Rev. D **53**, 3099 (1996), hep-th/9510029
15. W. Kummer, D.J. Schwarz, Nucl. Phys. B **382**, 171 (1992); H. Grosse et al., J. Math. Phys. **33**, 3892 (1992), hep-th/9205071

16. W. Kummer, P. Widerin, Phys. Rev. D **52**, 6965 (1995), gr-qc/9502031; W. Kummer, G. Tieber, Phys. Rev. D **59**, 044001 (1999), hep-th/9807122
17. D. Grumiller, W. Kummer, Phys. Rev. D **61**, 064006 (2000), gr-qc/9902074
18. H. Balasin, H. Nachbagauer, Class. Quant. Grav. **11**, 1453 (1994), gr-qc/9312028
19. M.O. Katanaev, I.V. Volovich, Phys. Lett. B **175**, 413 (1986); Ann. Phys. **197**, 1 (1990)
20. D.V. Vassilevich, A. Zelnikov, Nucl. Phys. B **594**, 501 (2001), hep-th/0009084
21. D. Grumiller, Int. J. Mod. Phys. A **17**, 989 (2001), hep-th/0111138
22. R.F. Streater, A.S. Wightman, PCT, spin and statistics, and all that (Advanced book classics) (Addison-Wesley, Redwood City 1989)
23. D. Grumiller, D. Hofmann, W. Kummer, Mod. Phys. Lett. A **16**, 1597 (2001), gr-qc/0012026
24. G.T. Horowitz, D. Marolf, Phys. Rev. D **52**, 5670 (1995), gr-qc/9504028; R.M. Wald, J. Math. Phys. **21**, 2802 (1980)
25. M.O. Katanaev, W. Kummer, H. Liebl, Phys. Rev. D **53**, 5609 (1996), gr-qc/9511009
26. A. Fabbri, J.G. Russo, Phys. Rev. D **53**, 6995 (1996), hep-th/9510109
27. Y. Uehara, Virtual black holes at linear colliders, hep-ph/0205068
28. S.B. Giddings, S. Thomas, Phys. Rev. D **65**, 056010 (2002), hep-ph/0106219; S. Dimopoulos, G. Landsberg, Phys. Rev. Lett. **87**, 161602 (2001), hep-ph/0106295
29. D. Grumiller, W. Kummer, D.V. Vassilevich, Positive specific heat of the quantum corrected dilaton black hole, hep-th/0305036